AD-A256 579

UNCLASSIFIED

(2)

AR-006-986

ELECTRONICS RESEARCH LABORATORY

Communications Division

RESEARCH REPORT
ERL-0643-RR

AN ANALYSIS OF THE EFFECT OF ACOUSTIC NON-LINEARITIES IN WIDE BANDWIDTH ACOUSTO-OPTIC BRAGG CELLS

by

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ABSTRACT (U)

In this report an extension of the theory of acoustic non-linearities accurate to fourth order is derived. A new fundamental limitation on the high frequency operation of acousto-optic signal processing architectures is demonstrated.

32 pp

AUG 92

APPROVED FOR PUBLIC RELEASE

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1 INTRODUCTION

Exploitation of the phenomena of acousto-optic diffraction to construct various real-time signal processing architectures is a familiar principle and a well established technology.

The ultimate microwave performance of these devices (in terms of achieving gigahertz operating frequencies, large bandwidths and high resolution) has been severely limited by exponential attenuation of the signal due to scattering of the coherent signal phonons by incoherent thermal phonons in the acousto-optic material. The room temperature ω^2 frequency dependence of the attenuation coefficient typically restricts usable performance to a maximum frequency of less than 3 GHz, with a 1.2 GHz bandwidth.

Recent research into the characteristics of cryogenically cooled Bragg deflectors [1, 2] has shown that efficient operation of these devices at frequencies well into the gigahertz regime is possible. During the course of the cryogenic programme it was found that the non-linear response of the acousto-optic material could cause significant power-dependent depletion of the acoustic signal due to the generation of higher harmonics [3].

In this report the theory of harmonic generation due to acoustic non-linearities is described. The general theory, accurate to fourth order in the non-linearity, is derived in Section 2. Various simplifications of the general theory are given in Section 3, along with example calculations. In Section 4 the conclusions of the report and its implications for the construction of wide bandwidth, cryogenically cooled acousto-optic signal processing systems are given. In Appendix A the sufficient conditions for employing a linearised definition of the acoustic strain field are derived.

2 THEORY OF ACOUSTIC NON-LINEARITIES

The acoustic field propagating in the crystal results in a particle displacement defined by

$$u = x_o - x_s \tag{1}$$

where x_o is the position of the particle in its rest state and x_s is the coordinate under the influence of the applied strain field. It has been shown [4] that for the case of a purely longitudinal mode, the equation of motion for the particle displacement is given by

$$\rho_o \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}\left[M_2 + M_3 \frac{\partial u}{\partial x} + M_4 \left(\frac{\partial u}{\partial x}\right)^2 + \ldots\right] \tag{2}$$

where ρ_o is the density of the crystal in the unstressed condition and x is the Lagrangian coordinate in the direction of acoustic propagation. Equation (2) assumes that all harmonics generated are in the same direction as the fundamental and are also purely longitudinal

modes. The M_i are non-linear coefficients, which are combinations of elastic coefficients up to and including order i [4].

Acoustic measurements undertaken by Signal Processing Group rely on exploiting acousto-optic diffraction of a laser beam from the acoustic signal to obtain information about the behaviour of the acoustic field [1, 2]. The intensity of the diffracted light is proportional to the intensity of the acoustic strain field. As such, for the purpose of comparing the final theoretical results with experimental measurements, it is useful to convert Equation (2) into an expression involving the one dimensional acoustic strain, defined by [5]

$$S \equiv \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2 \tag{3}$$

It is shown in Appendix A that for the case where the non-linearity is dominated by material parameters satisfying $M_2 < M_3 < M_4 < \ldots$ etc, it is permissible to retain only the linear contribution from Equation (3). Substituting this into Equation (2) gives

$$\rho_0 \frac{\partial^2 S}{\partial t^2} = M_2 \frac{\partial^2 S}{\partial x^2} + M_3 \frac{\partial}{\partial x}\left(S\frac{\partial S}{\partial x}\right) + M_4 \frac{\partial}{\partial x}\left(S^2 \frac{\partial S}{\partial x}\right) + \ldots \tag{4}$$

The first term on the right hand side is just the usual Hooke's Law for one-dimensional motion. The higher-order harmonics arise as a result of the second, third, etc terms in Equation (4). Assuming a dispersionless medium, the total strain field resulting from N harmonics can be written

$$S_T = \frac{1}{2}\sum_{p=1}^{N}\left(S_{p\omega}\, e^{pj(\omega t - kx)} + S^*_{p\omega}\, e^{-pj(\omega t - kx)}\right) \tag{5}$$

where $S_{p\omega}$ is the complex strain envelope at frequency $p\omega$ and pk is the corresponding wavenumber. It is convenient to introduce the shorthand notation

$$S_T = \frac{1}{2}\sum_{p=1}^{N}\left(S_p\, e^{pj} + S^*_p\, e^{-pj}\right) \tag{6}$$

where the term $\exp(\omega t\text{-}kx)$ is understood.

Following the procedure of Elston and Kellman [5], equation (6) is substituted into Equation (4) and terms of equal exponential phase and frequency are identified. Taking the time derivatives of Equation (6) yields for the left hand side of Equation (4)

$$\frac{\partial^2 S_T}{\partial t^2} = \frac{-1}{2}\sum_{p=1}^{N}\left((p\omega)^2 S_p\, e^{pj} + \text{c.c.}\right) \tag{7}$$

The spectral component at frequency $p\omega$ is thus

$$\frac{\partial^2 S_p}{\partial t^2} = \frac{-1}{2}(p\omega)^2 S_p e^{pj} \tag{8}$$

Higher order terms in $\frac{\partial^n S_p}{\partial t^n}$ have been set to zero on the right hand side of Equation (7). In carrying out the spatial derivatives, it is convenient to initially calculate only the simple harmonic (elastic) contribution from the first term on the right hand side of Equation (4). This is

$$\frac{\partial^2 S_T}{\partial x^2} = \frac{-1}{2}\sum_{p=1}^{N}\left(\frac{\partial^2 S_p}{\partial x^2} e^{pj} - 2kpj\frac{\partial S_p}{\partial x} e^{pj} - (kp)^2 S_p e^{pj} + \text{c.c.}\right) \tag{9}$$

The spatial derivatives are now simplified by invoking the slowly-varying envelope approximation (SVEA) i.e.

$$k^2 S_p \gg k\frac{\partial S_p}{\partial x} \gg \frac{\partial^2 S_p}{\partial x^2} \tag{10}$$

which gives for the $p\omega$ component,

$$\frac{\partial^2 S_p}{\partial x^2} = -\left(kpj\frac{\partial S_p}{\partial x} + \frac{(kp)^2}{2} S_p\right)e^{pj} \tag{11}$$

Substituting equations (8) and (11) into equation (4) yields for the normal elastic contribution

$$-\rho_0\frac{(p\omega)^2}{2} S_p = -M_2\left(\frac{(kp)^2}{2} S_p + kpj\frac{\partial S_p}{\partial x}\right) + \{\text{higher order non-linear terms}\} \tag{12}$$

As previously noted the coefficient M_2 is the usual Hooke's Law elastic coefficient, and is related to ω, ρ_0, k and the acoustic phase velocity V_P via the relations

$$V_P = \frac{\omega}{k} = \left(\frac{M_2}{\rho_0}\right)^{1/2} \tag{13}$$

Equation (12) can therefore be expressed as

$$\frac{\partial S_p}{\partial x} = \frac{-j\omega}{\rho_0 k^2 p V_p^3} \times \{\text{higher order non-linear terms}\} \tag{14}$$

Having exploited the simplification arising through the determination of the harmonic contribution, it is appropriate to turn to the evaluation of the non-linear contributions. Invoking the SVEA again yields expressions for the higher order terms given by

$$\frac{\partial S_T}{\partial x} = -\frac{1}{2}\sum_{m=1}^{N} mjk\left(S_m\, e^{mj} - S_m^*\, e^{-mj}\right) \tag{15}$$

$$S_T\frac{\partial S_T}{\partial x} = -\frac{1}{4}\sum_{m=1}^{N}\sum_{n=1}^{N} mjk\left(S_m\, S_n\, e^{(m+n)j} - S_m^*\, S_n\, e^{(n-m)j} + S_m\, S_n^*\, e^{(m-n)j} - S_m^*\, S_n^*\, e^{-(m+n)j}\right) \tag{16}$$

$$\begin{aligned} S_T^2\frac{\partial S_T}{\partial x} = -\frac{1}{8}\sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{r=1}^{N} mjk\Big(& S_mS_nS_r e^{(m+n+r)j} - S_m^*S_nS_r\, e^{(n+r-m)j} + S_mS_n^*S_r\, e^{(m+r-n)j} \\ & - S_m^*S_n^*S_r\, e^{(r-m-n)j} + S_mS_nS_r^*\, e^{(m+n-r)j} - S_m^*S_nS_r^*\, e^{(n-m-r)j} + S_mS_n^*S_r^*\, e^{(m-n-r)j} \\ & - S_m^*S_n^*S_r^*\, e^{-(m+n+r)j}\Big) \end{aligned} \tag{17}$$

Differentiating Equations (15), (16) and (17) under the constraints of the SVEA, and substituting the result into equation (14) yields

$$\begin{aligned} \frac{\partial S_p}{\partial x} = -\frac{-j\omega}{4\rho_o V_p^3}\Big[& M_3\sum_{m=1}^{N}\sum_{n=1}^{N} m\left(S_mS_n\,\delta[p-(m+n)] - S_m^*S_n\,\delta[p-(n-m)] + S_mS_n^*\,\delta[p-(m-n)]\right) \\ & +\frac{M_4}{2}\sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{r=1}^{N} m\big(S_mS_nS_r\,\delta[p-(m+n+r)] - S_m^*S_nS_r\,\delta[p-(n+r-m)] \\ & + S_mS_n^*S_r\,\delta[p-(m+r-n)] - S_m^*S_n^*S_r\,\delta[p-(r-m-n)] + S_mS_mS_r^*\,\delta[p-(m+n-r)] \\ & - S_m^*S_nS_r^*\,\delta[p-(n-m-r)] + S_mS_n^*S_r^*\,\delta[p-(m-n-r)]\big)\Big] \end{aligned} \tag{18}$$

The delta functions express the frequency matching constraints on harmonics m, n and r, and can of course be used to kill one of the summations in each term if required. It is convenient however to retain these summations for the moment. These constraints, for the physically meaningful solutions with positive ω, preclude inclusion of terms such as $S_m^*S_n^*$ and $S_m^*S_n^*S_r^*$. The exponential frequency term $\exp(p\omega)$ that must appear on both sides of Equation (18) is understood.

In order to clarify the underlying physics of Equation (18), the complex strain field S_p is expressed in terms of the real amplitude and phase via

$$S_p \equiv \widehat{S}_p \exp(j\psi_p) \tag{19}$$

Substituting Equation (19) into Equation (18) and separating the result into the real and imaginary parts yields (dropping the caretes for simplicity of notation)

$$\frac{\partial S_p}{\partial x} = -\gamma\omega M_3 \sum_{m=1}^{N}\sum_{n=1}^{N} mS_mS_n \left(\sin(\psi_m+\psi_n-\psi_p)\,\delta[p-(m+n)] - \sin(\psi_n-\psi_m-\psi_p)\,\delta[p-(n-m)]\right.$$
$$\left. + \sin(\psi_m-\psi_n-\psi_p)\,\delta[p-(m-n)]\right)$$
$$-\frac{\gamma\omega M_4}{2}\sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{r=1}^{N} m\,S_mS_nS_r\left(\sin(\psi_m+\psi_n+\psi_r-\psi_p)\,\delta[p-(m+n+r)]\right.$$
$$- \sin(\psi_n+\psi_r-\psi_m-\psi_p)\,\delta[p-(n+r-m)] + \sin(\psi_m+\psi_r-\psi_n-\psi_p)\,\delta[p-(m+r-n)] \qquad (20)$$
$$- \sin(\psi_r-\psi_m-\psi_n-\psi_p)\,\delta[p-(r-m-n)] + \sin(\psi_m+\psi_n-\psi_r-\psi_p)\,\delta[p-(m+n-r)]$$
$$\left.- \sin(\psi_n-\psi_m-\psi_r-\psi_p)\,\delta[p-(n-m-r)] + \sin(\psi_m-\psi_n-\psi_r-\psi_p)\,\delta[p-(m-n-r)]\right)$$

where

$$\gamma \equiv \frac{1}{4\rho_O V_p^3}\,. \qquad (21)$$

Equation (20) describes the evolution of the envelope of the strain field. The corresponding equation for the imaginary terms is

$$0 = M_3 \sum_{m=1}^{N}\sum_{n=1}^{N} mS_mS_n \left(\cos(\psi_m+\psi_n-\psi_p)\,\delta[p-(m+n)] - \cos(\psi_n-\psi_m-\psi_p)\,\delta[p-(n-m)]\right.$$
$$\left. + \cos(\psi_m-\psi_n-\psi_p)\,\delta[p-(m-n)]\right)$$
$$+\frac{M_4}{2}\sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{r=1}^{N} m\,S_mS_nS_r\left(\cos(\psi_m+\psi_n+\psi_r-\psi_p)\,\delta[p-(m+n+r)]\right.$$
$$- \cos(\psi_n+\psi_r-\psi_m-\psi_p)\,\delta[p-(n+r-m)] + \cos(\psi_m+\psi_r-\psi_n-\psi_p)\,\delta[p-(m+r-n)] \qquad (22)$$
$$- \cos(\psi_r-\psi_m-\psi_n-\psi_p)\,\delta[p-(r-m-n)] + \cos(\psi_m+\psi_n-\psi_r-\psi_p)\,\delta[p-(m+n-r)]$$
$$\left.- \cos(\psi_n-\psi_m-\psi_r-\psi_p)\,\delta[p-(n-m-r)] + \cos(\psi_m-\psi_n-\psi_r-\psi_p)\,\delta[p-(m-n-r)]\right)$$

Equations (20) and (22) are the final expressions for the evolution of the magnitude of the envelope of the strain field at frequency $p\omega$ as a function of propagation distance x, accurate to fourth order in the non-linearity. The extension of the equations to fifth, sixth and higher order non-linearities is obvious.

Equation (22) constrains the contributing terms to have phases that combine such that their sum is always $\theta_k = \pm\left(k+\frac{1}{2}\right)\pi$, k = 0, 1, 2, .. . This demonstrates the so-called "phase-locking" phenomena of the harmonics that has been previously noted[6]. Having established all possible terms that may contribute to the evolution of the spectral component S_p of the strain (as determined by the delta functions and equation (20)), the phase constraints then select those terms that, in effect, ensure conservation of energy of the total system. The phase locking implies that the sine terms of Equation (20) must all be constrained to a value of either +1 or -1. As will be shown explicitly in the following example, a choice of +1 for these

terms leads to solutions that satisfy conservation of acoustic power and are intuitively correct. Equation (20) can thus be written as

$$\frac{\partial S_p}{\partial x} = -\gamma\omega M_3 \sum_{m=1}^{N}\sum_{n=1}^{N} m S_m S_n \left(\delta[p-(m+n)] - \delta[p-(n-m)] + \delta[p-(m-n)]\right)$$

$$- \frac{\gamma\omega M_4}{2} \sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{r=1}^{N} m S_m S_n S_r \left(\delta[p-(m+n+r)] - \delta[p-(n+r-m)] + \delta[p-(m+r-n)]\right. \tag{23}$$

$$\left. - \delta[p-(r-m-n)] + \delta[p-(m+n-r)] - \delta[p-(n-m-r)] + \delta[p-(m-n-r)]\right)$$

In order to demonstrate conservation of acoustic power of the general equations and the validity of the choice of sign for the phase terms, Equation (20) will be used in the following example.

<u>Example: fourth order non-linearity, N=3</u>

The contributing terms are found to be

$$\frac{\partial S_1}{\partial x} = -\gamma\omega M_3\left(S_1S_2 \sin\xi_1 + S_2S_3 \sin\xi_2\right) - \frac{\gamma\omega M_4}{2}\left(S_1^2S_3 \sin\xi_3 + S_2^2S_3 \sin\xi_4\right) \tag{24}$$

$$\frac{\partial S_2}{\partial x} = \gamma\omega M_3\left(S_1^2 \sin\xi_1 - 2S_1S_3 \sin\xi_2\right) + 2\gamma\omega\, M_4\, S_1S_2S_3 \sin\xi_4 \tag{25}$$

$$\frac{\partial S_3}{\partial x} = 3\gamma\omega M_3\, S_1S_2 \sin\xi_2 - \frac{\gamma\omega M_4}{2}\left(3S_1S_2^2 \sin\xi_4 - S_1^3 \sin\xi_3\right) \tag{26}$$

where the ξ_i are the combinations of phase terms appearing in Equations (20) and (22).

Conservation of power requires

$$S_1^2 + S_2^2 + S_3^2 = \text{constant} \tag{27}$$

which implies that

$$S_1\frac{\partial S_1}{\partial x} + S_2\frac{\partial S_2}{\partial x} + S_3\frac{\partial S_3}{\partial x} = 0 \tag{28}$$

Substituting Equations (24) to (26) into Equation (28) confirms that Equations (24) to (26) satisfy conservation of acoustic energy independent of the choice of +1 or -1 for the value of the sine terms. Choosing the sine terms to have a value of +1 is intuitively reasonable since (i) this leads to an overall negative derivative for the fundamental indicating that, at least initially, the fundamental will suffer depletion (as it must), and (ii) terms with m+n=p are always associated with a positive derivative, as expected for lower frequency contributions that "pump" the higher harmonic. These considerations finally give a set of equations for the evolution of the three harmonics, accurate to fourth order in the non-linearity, as

$$\frac{\partial S_1}{\partial x} = -\gamma\omega M_3\left(S_1S_2 + S_2S_3\right) - \frac{\gamma\omega M_4}{2}\left(S_1^2S_3 + S_2^2S_3\right) \tag{29}$$

$$\frac{\partial S_2}{\partial x} = \gamma\omega M_3\left(S_1^2 - 2S_1S_3\right) + 2\gamma\omega M_4S_1S_2S_3 \tag{30}$$

$$\frac{\partial S_3}{\partial x} = 3\gamma\omega M_3S_1S_2 - \frac{\gamma\omega M_4}{2}\left(3S_1S_2^2 - S_1^3\right) \tag{31}$$

3 SIMPLIFICATION OF THE GENERAL THEORY

In this section several examples are given for which the specific form of the rate equations which describe harmonic generation due to non-linear acoustic interactions can be simplified under various approximations . The examples given are

(i) the rate equations and analytic solution for the third order, two harmonic case

(ii) a simplified form of the rate equations for the third order, N harmonic case

(iii) parameterisation of the full solution for low to moderate acoustic powers

Acoustic attenuation is included in a phenomenological manner in each of the cases as a simple additional term in the rate equations. For the purpose of the calculations the L[001] propagation mode of TeO_2 at a frequency of 1 GHz is used as the standard example, and an ω^2 frequency dependence is assumed. A transducer with radius 0.125 mm is used throughout and the standard input power for all calculations is 15 mW unless otherwise specified.

The L[001] mode of TeO_2 exhibits an anomalously large acoustic non-linearity due to a weak oxygen-oxygen bond in the lattice [7], and is therefore ideally suited to the study of acoustic non-linear phenomena. Furthermore, despite the fact that TeO_2 is piezoelectric, the L[001] propagation mode is not effected [8] and hence there is no piezoelectric stiffening of the elastic constants of the material.

Numerical solution of the coupled differential equations that occur in examples (i) and (ii) was undertaken using a fourth order Runge-Kutta method.

(i) Third order, two harmonic case:

In this case the rate equations simplify to

$$\frac{\partial S_1}{\partial x} = -\alpha_1 S_1 - M_3\gamma\omega S_1 S_2 \tag{32}$$

$$\frac{\partial S_2}{\partial x} = -\alpha_2 S_2 + M_3 \gamma \omega S_1^2 \tag{33}$$

where $\alpha_{1,2}$ denotes the acoustic absorption coefficient in Nepers/m for frequency $\omega_{1,2}$. Equations (32) and (33) can be solved analytically for the case $\alpha_{1,2} = 0$, resulting in

$$S_1(x) = S_1(0)\,\mathrm{sech}(\gamma\omega S_1(0) M_3 x) \tag{34}$$

$$S_2(x) = S_1(0)\,\tanh(\gamma\omega S_1(0) M_3 x) \tag{35}$$

The strain is related to acoustic power P_a by

$$P_a = \frac{\rho_O V_P^3 S^2 A}{2} \tag{36}$$

where A is the cross sectional area of the acoustic beam. Thus, Equations (34) and (35) can be written in terms of the initial acoustic power in the fundamental, P_O, as

$$P_1(x) = P_O \mathrm{sech}^2\left(\Gamma\sqrt{P_O/A}\,\omega x\right) \tag{37}$$

$$P_2(x) = P_O \tanh^2\left(\Gamma\sqrt{P_O/A}\,\omega x\right) \tag{38}$$

where the *harmonic generation constant* [5], Γ, is defined by

$$\Gamma = \frac{M_3}{(2\rho_O)^{1.5} V_P^{4.5}} \tag{39}$$

Figure 1 demonstrates second harmonic generation due to the third order non-linearity of the L[001] mode of TeO_2, for a frequency of 1 GHz. From Equations (37), (38) and (39) it is apparent that non-linear effects will be important for Bragg cells that operate at high frequencies and have slow phase velocities (and hence large time-bandwidth products) - unfortunately the very properties that are highly desirable for EW/SIGINT applications. Figure 2 shows the numerical solution of Equations (32) and (33) for the same parameters as in Figure 1 but including an attenuation of 0.5 dB/mm, assuming an ω^2 frequency dependence for the attenuation coefficient. The effect of attenuation is clearly to decrease the coupling of the fundamental to the harmonic, and extend the distance over which significant power can be found in the fundamental. This suggests that in systems dominated by acoustic nonlinearities a small amount of frequency dependent attenuation is in fact beneficial to the propagation characteristics of the fundamental.

(ii) Third order non-linearity, N harmonic case:

Probably the easiest method to obtain the general form of the rate equations for an arbitrary number of frequencies is that of writing down the equations for the first M harmonics and

generalising from the observed symmetry of the resulting coupled differential equations. For example, taking the fundamental and first five harmonics yields

$$\frac{\partial S_1}{\partial x} = -\alpha_1 S_1 - \gamma\omega M_3(S_1S_2 + S_2S_3 + S_3S_4 + S_4S_5 + S_5S_6) \tag{40}$$

$$\frac{\partial S_2}{\partial x} = -\alpha_2 S_2 + 2\gamma\omega M_3\left(\frac{S_1^2}{2} - S_1S_3 - S_2S_4 - S_3S_5 - S_4S_6\right) \tag{41}$$

$$\frac{\partial S_3}{\partial x} = -\alpha_3 S_3 + 3\gamma\omega M_3(S_1S_2 - S_1S_4 - S_2S_5 - S_3S_6) \tag{42}$$

$$\frac{\partial S_4}{\partial x} = -\alpha_4 S_4 + 4\gamma\omega M_3\left(S_1S_3 + \frac{S_2^2}{2} - S_1S_5 - S_2S_6\right) \tag{43}$$

$$\frac{\partial S_5}{\partial x} = -\alpha_5 S_5 + 5\gamma\omega M_3(S_1S_4 + S_2S_3 - S_1S_6) \tag{44}$$

$$\frac{\partial S_6}{\partial x} = -\alpha_6 S_6 + 6\gamma\omega M_3\left(S_1S_5 + S_2S_4 + \frac{S_3^2}{2}\right) \tag{45}$$

Ignoring the loss terms, the symmetry of these equations clearly demonstrates the following rules for writing down the rate equation describing the evolution of the harmonic at frequency $p\omega$. These are, for each term S_mS_n(noting that $m \leq n$ always)

(1) if $n < p$ then the term is $+S_mS_n$

(2) if $n > p$ then the term is $-S_mS_n$

(3) if $m = n$ then the term is $+\frac{S_m^2}{2}$

These observations lead to the general third order expression for the harmonic at frequency $p\omega$ as

$$\frac{\partial S_p}{\partial x} = -\alpha_p S_p + p\gamma\omega M_3\left((1-\delta_{1p})\sum_{m=1}^{L}\frac{S_mS_{N-m}}{1+\delta_{tm}} - (1-\delta_{Np})\sum_{m=1}^{N-p} S_mS_{m+p}\right) \tag{46}$$

where the δ_{ij} are Kronecker deltas, $t = \frac{p}{2}$ (only integer for p even) and L = TRUNC(t), where TRUNC means "truncate to integer".

Figure 3 shows the behaviour of the fundamental calculated from the numerical solution for the L[001] mode for the cases of N = 2, 10, 20 and 40 harmonics and no attenuation. It is apparent that even in the relatively short space of a typical Bragg cell aperture, the effect of the very high harmonics on the behaviour of the fundamental is substantial, indicating that under low loss conditions accurate modelling of the fundamental becomes very involved.

Another interesting feature of Figure 3 is that the energy does not completely drain from the fundamental into higher harmonics, as is suggested by the results for N=2. In fact, as shown in Figure 4, after a sufficiently large propagation distance a dynamic equilibrium of the distribution of acoustic power between the N modes is achieved. Figure 4 shows the full numerical solution of Equations (40) to (45) for the case of zero attenuation.

The effect of an acoustic attenuation of 0.002 dB/mm is demonstrated in Figure 5. As expected the assumed ω^2 dependence of the attenuation very significantly reduces the influence of higher harmonics, and it is interesting to note that the oscillations in the fundamental are not only damped but the periodicity is also slightly altered in the presence of finite attenuation.

It is clear from these results that any acousto-optic signal processing device relying on a cryogenically cooled TeO_2 Bragg cell operating in the GHz regime would have its performance severely effected by acoustic non-linearities. This is expected to also be true of materials such as GaP and $LiNbO_3$, which could begin to exhibit similar behaviour at around the 10 GHz region (the non-linear coefficients of these materials are about an order of magnitude smaller than those of the L[001] mode of TeO_2 [5], and as such will only play a role at higher frequencies. At such high frequencies however, dispersion effects could significantly modify the non-linear response).

This degradation in the high frequency performance of acousto-optic devices is intrinsic to the physical nature of the material and as such, unlike the deleterious effects of thermal attenuation, represents a fundamental limitation to the performance of these devices in the GHz regime.

One possibility for operating acousto-optic devices at multi-gigahertz frequencies as suggested by Figures (4) and (5) may be to exploit the region in which there is a dynamic equilibrium between the harmonics. This could be achieved by constructing longer cells (an option made feasible by the negligible attenuation at cryogenic temperatures), or by operating at high frequencies or high powers (the latter options essentially "compress" the highly oscillatory region). The effects of acoustic non-linearities may then be no more troublesome than the more familiar near-field *integrated optical effect* [9], which causes anomalous diffraction of light in regions very near the transducer. Thus, for example, by exploiting the region of dynamic equilibrium it may be possible to construct a very high frequency acousto-optic deflector suitable for mode-locking diode lasers or for noise suppression applications. In most signal processing applications however, depletion of the fundamental due to the non-linear response of the acousto-optic material will fundamentally limit the performance of cryogenically cooled, high frequency devices.

(iii) Parameterised model for the non-linear behaviour:

In section 3(i) it was shown that the most fundamental non-linear interaction involves a sech^2 depletion of the fundamental to the second harmonic (Equation (37)). Thus, for low powers, it is reasonable to assume that the power depletion for the general case should initially be dominated by this sech^2 behaviour. From Figure (4) it is apparent that the major effect of

accounting for higher harmonics in the model is to allow acoustic energy to return to the fundamental, resulting in a slowing of the overall rate of depletion and a finite amount of energy always being present in the fundamental. From Figure (5), it can be seen that the effect of acoustic attenuation is to reduce this feedback and finally to dominate the decay characteristics for large attenuations or large propagation distances. Such observations suggest a very simple model for the decay of power in the fundamental, ie

$$P_1(x) = P_0 \mathrm{sech}^2\left(\Gamma\sqrt{P_0/A}\,\omega x\right) + P_B \qquad (47)$$

where the addition of the baseline P_B is the simplest method of retaining energy in the fundamental. Thus, the parametric model basically describes a sech^2 decay of the fundamental with the baseline (for which there is no justification in the exact theory) accounting for the influence of coupling to higher harmonics.

The result of fitting Equation (47) to the full theoretical curves is shown in Figures (6) and (7). An attenuation of 0.004 dB/mm (which corresponds roughly to a temperature of 4 K) has been used in the calculation of the exact solutions. Figure (8) shows a plot of the acoustic power based on the parametric model as a function of acoustic power used in the exact solutions. It is clear from these results that the parametric model describes the actual behaviour quite well for low to moderate acoustic powers using the indicated scaling parameters. This analytic model can therefore be used to describe the depletion of the fundamental for low to moderate acoustic powers and small attenuation, and provides a simple model for the behaviour of the fundamental in the presence of acoustic non-linearities. Such a model can be applied advantageously in characterising experimental data, where only the parameters Γ and P_0/Γ_B need be recorded.

4 CONCLUSIONS

In this report an extension of the third order theory of acoustic non-linearities based on the approach by Elston and Kellman [5] has been derived. The necessity of going to fourth order in the non-linear response was driven by the requirement for a theoretical model capable of describing the experimentally observed characteristics of the power dependent depletion of a signal propagating in a cryogenically cooled acousto-optic material exhibiting extremely low attenuation. These results have been reported elsewhere [3].

In Section 2 the general expression for the strain field accurate to fourth order in the non-linearity was derived.

In Section 3 several specific examples were given which highlighted simplified descriptions for the non-linear response, and a parametric model accurate for low to moderate acoustic powers was presented.

The results of the modelling indicate that power dependent depletion of the input signal due to harmonic generation will be a fundamental limitation to the utility of acousto-optic signal

processing architectures operating in the gigahertz regime. Acousto-optic materials with slow phase velocities capable of operating at high frequencies (i.e. those materials that have a large time-bandwidth product, which corresponds to high resolution) are the most severely affected.

Under certain circumstances, the region of dynamic equilibrium which is established between harmonics may find some use, however it is unlikely that these will include the very high resolution requirements of many EW/SIGINT signal processing applications.

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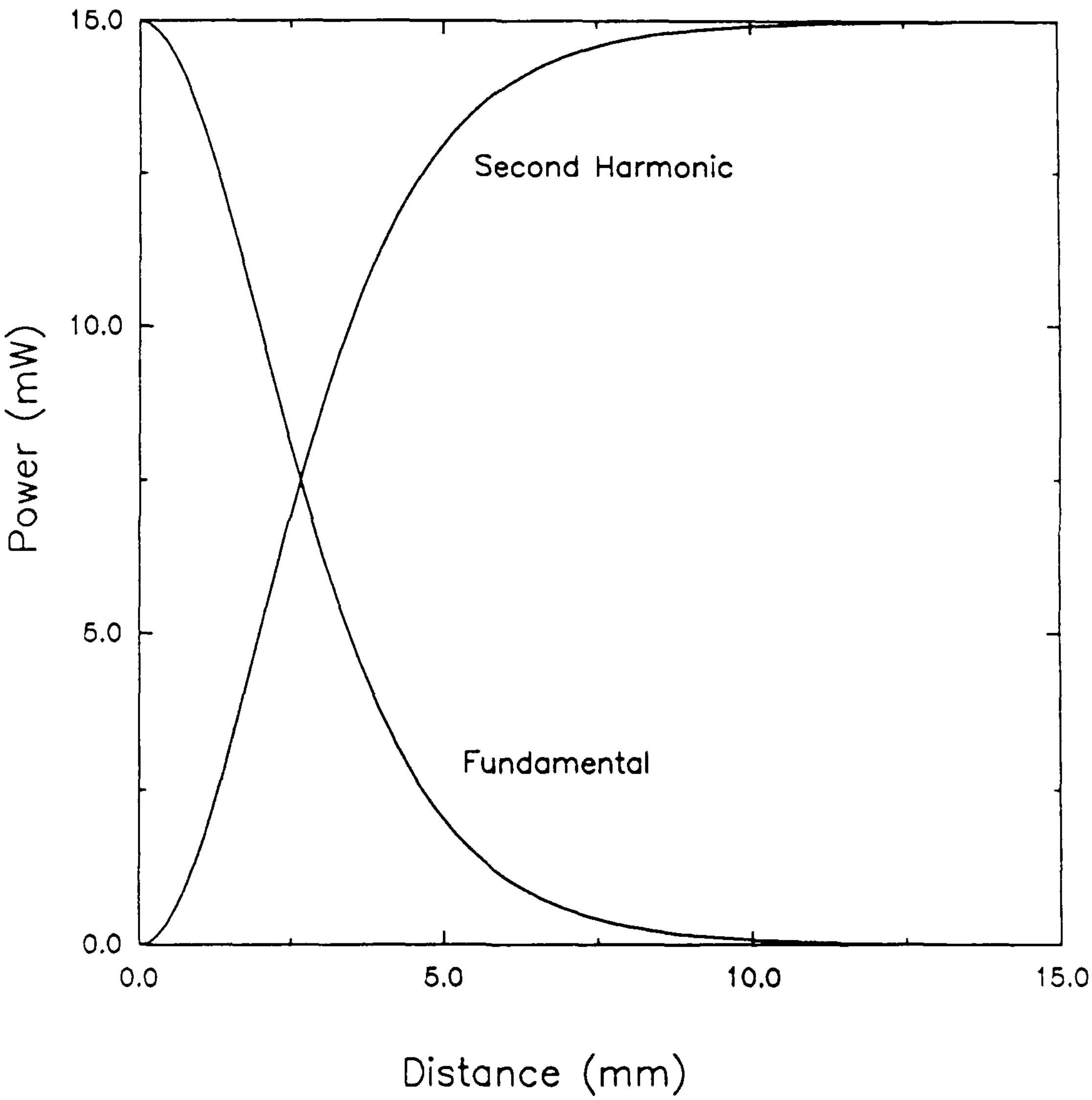

Figure 1 **Analytic solution of the two harmonic, third order non-linear depletion of the fundamental for the L[001] mode of TeO_2.**

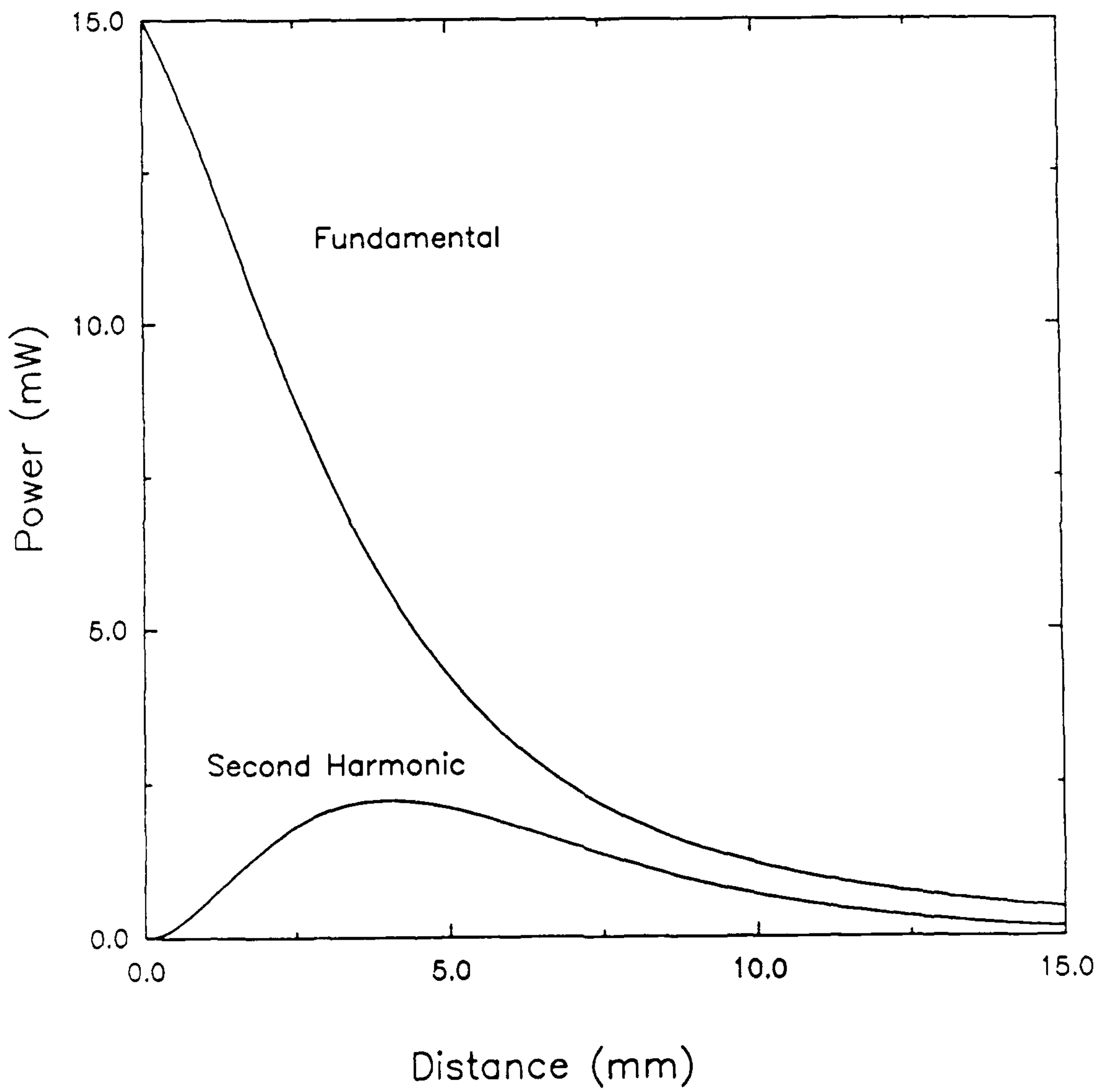

Figure 2 **Numerical solution of the two harmonic, third order non-linear depletion of the fundamental for the L[001] mode of TeO_2, including attenuation**

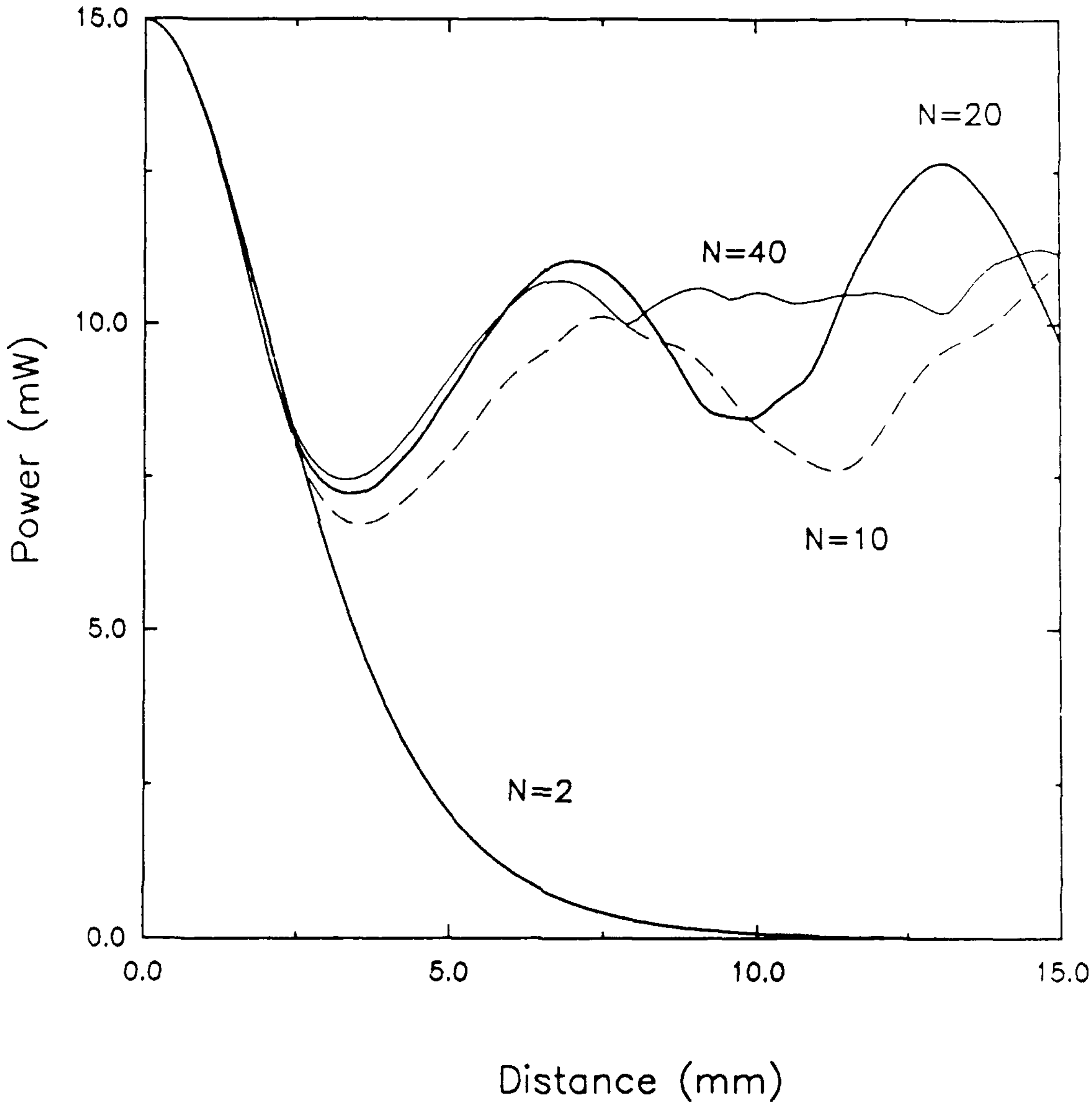

Figure 3 **Dependence of harmonic depletion on number of harmonics chosen to descibe the system. No attenuation.**

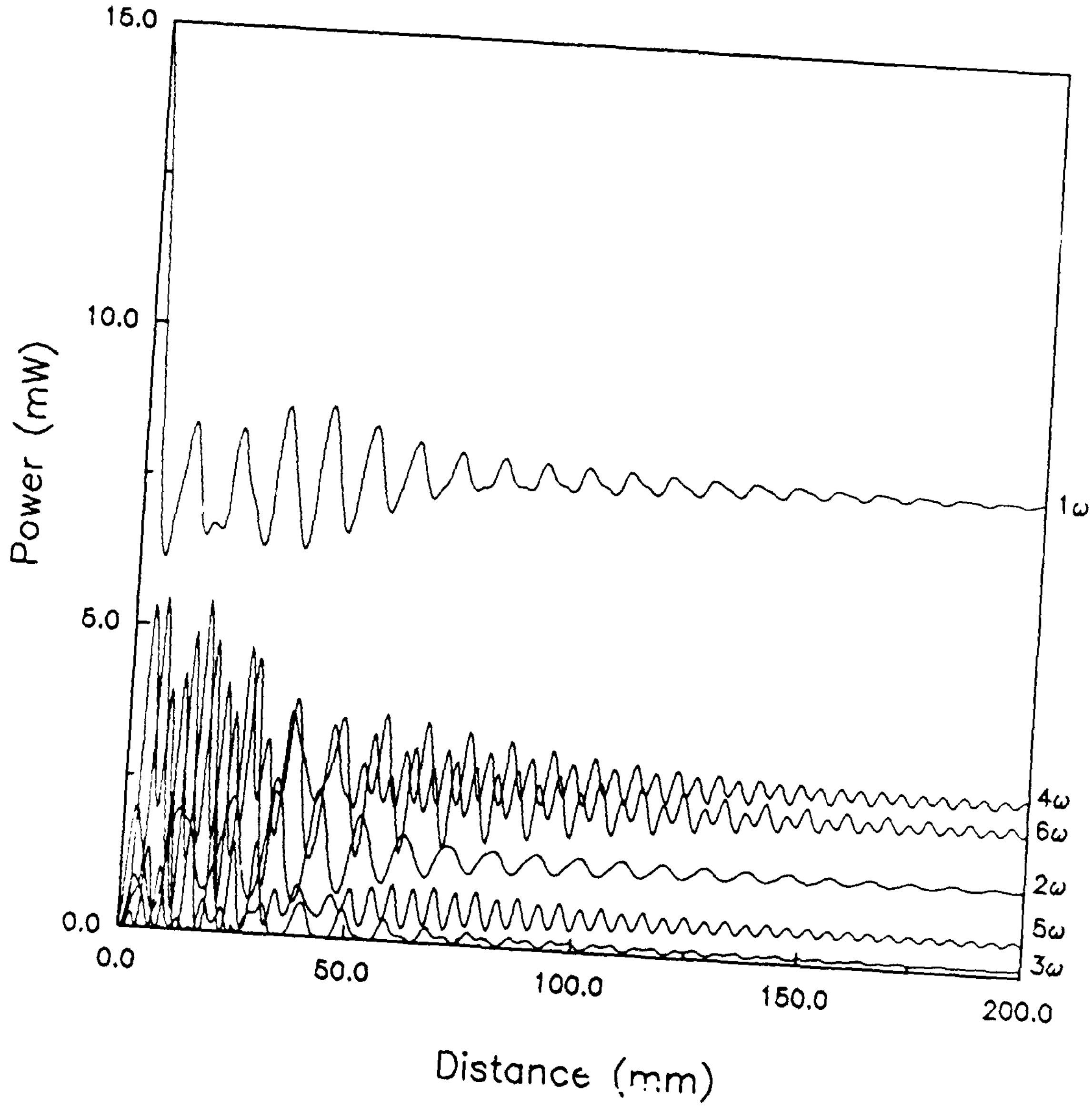

Figure 4 Demonstration of the evolution of dynamic equilibrium between the harmonics for sufficiently large propagation distances.

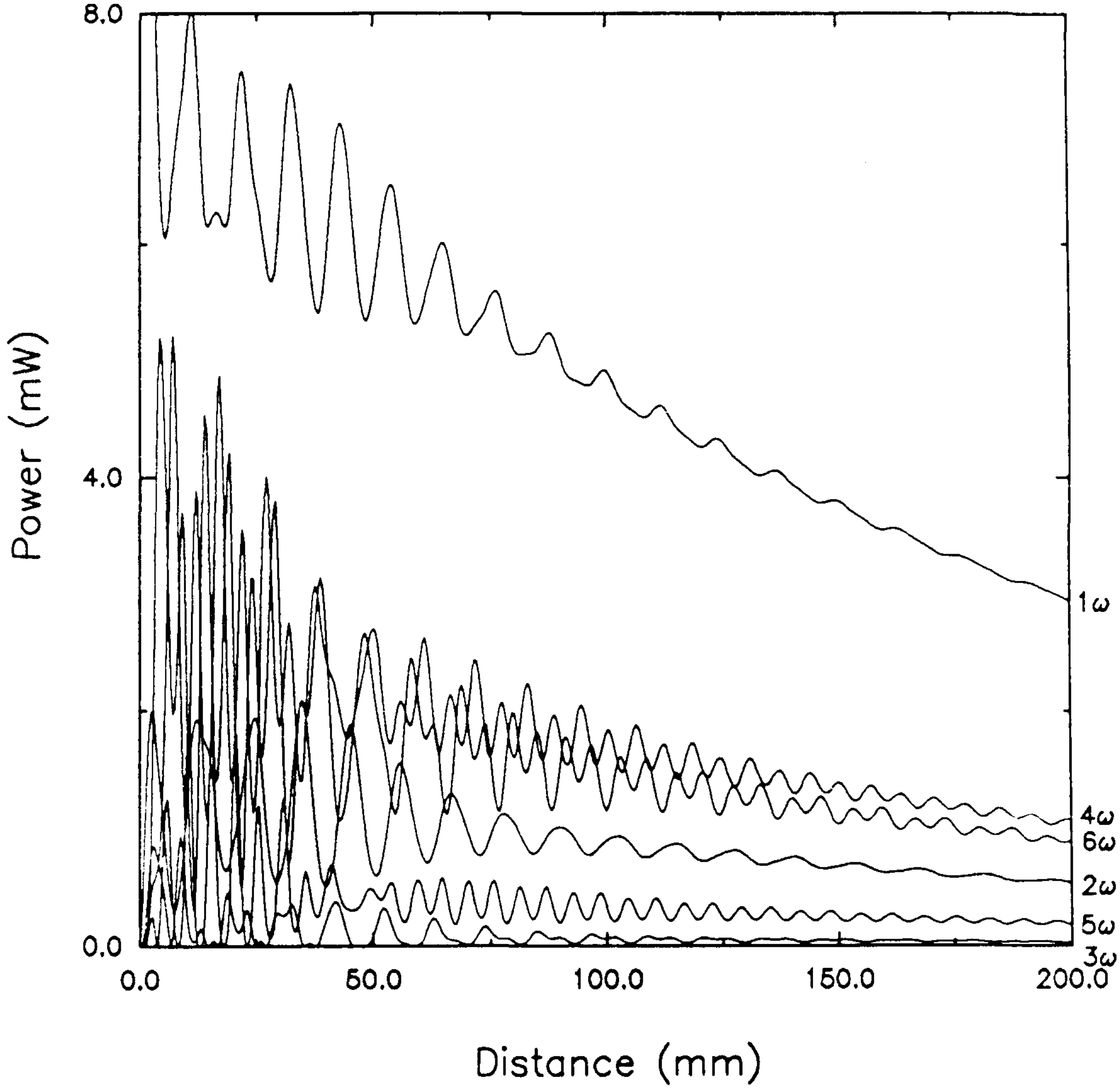

Figure 5 Demonstration of the evolution of dynamic equilibrium between the harmonics for sufficiently large propagation distances, but showing the effect of attenuation.

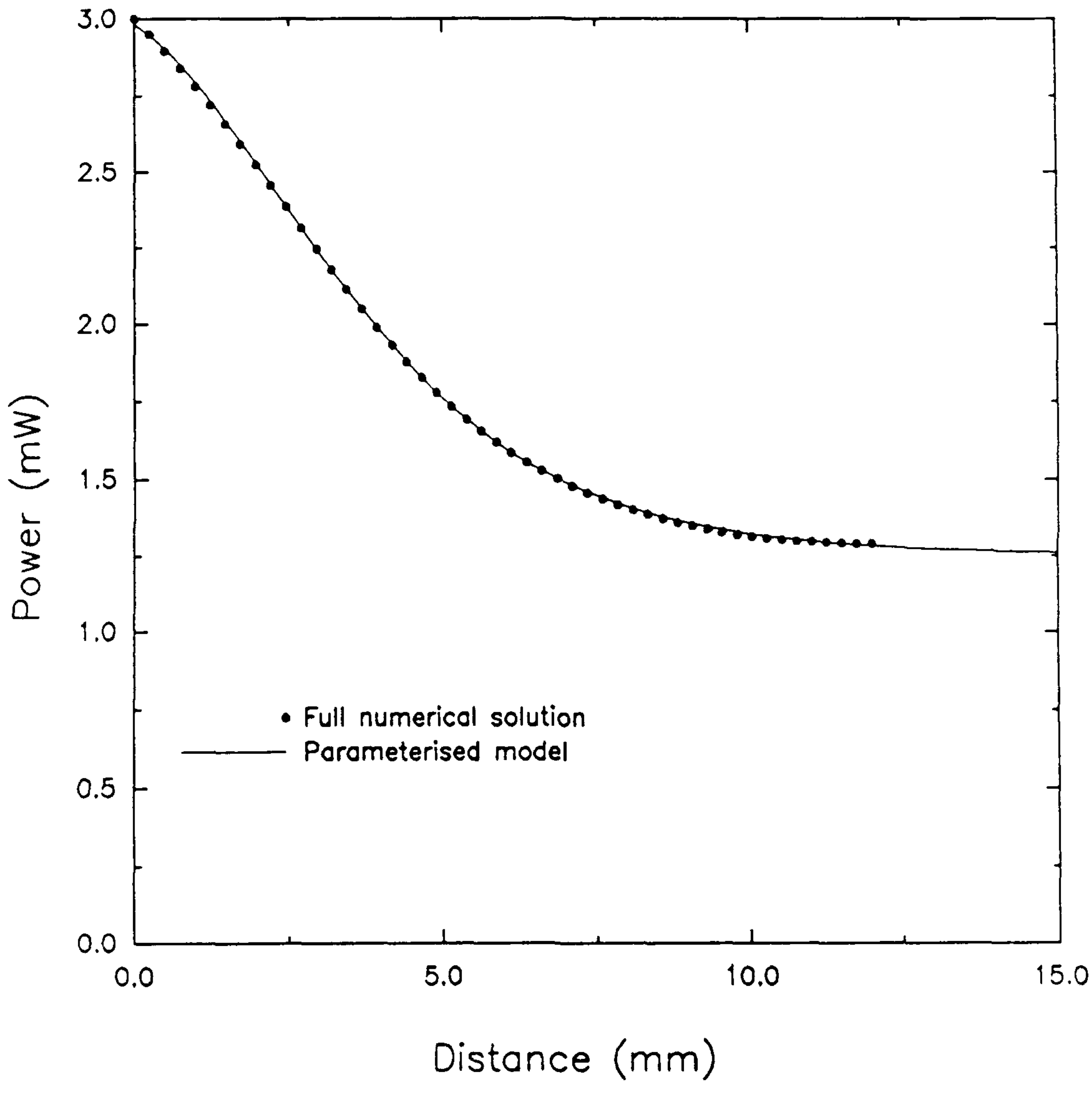

Figure 6 **Parameterised fit to full numerical solution for low acoustic input power.**

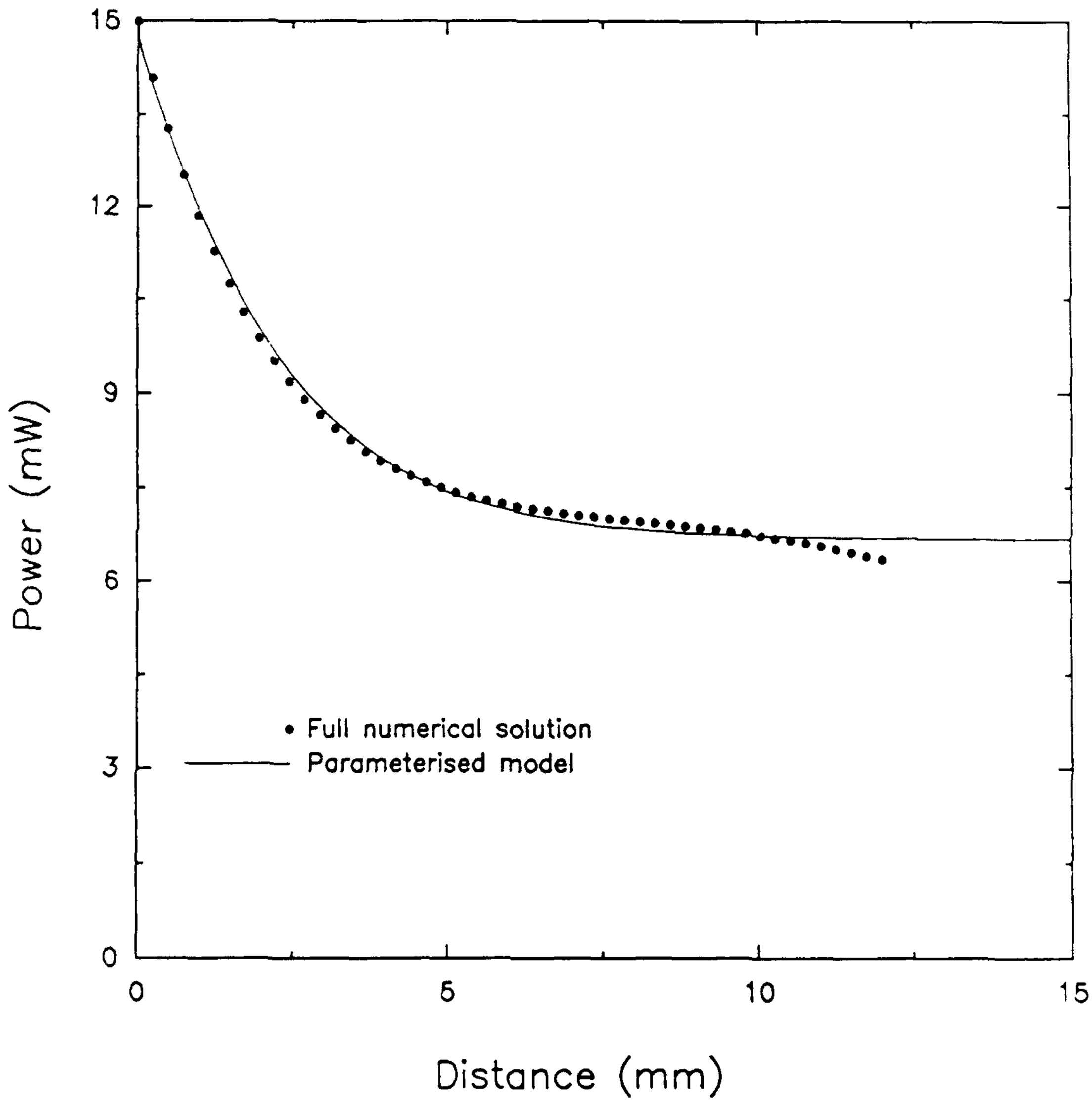

Figure 7 **Parameterised fit to full numerical solution for higher acoustic input power.**

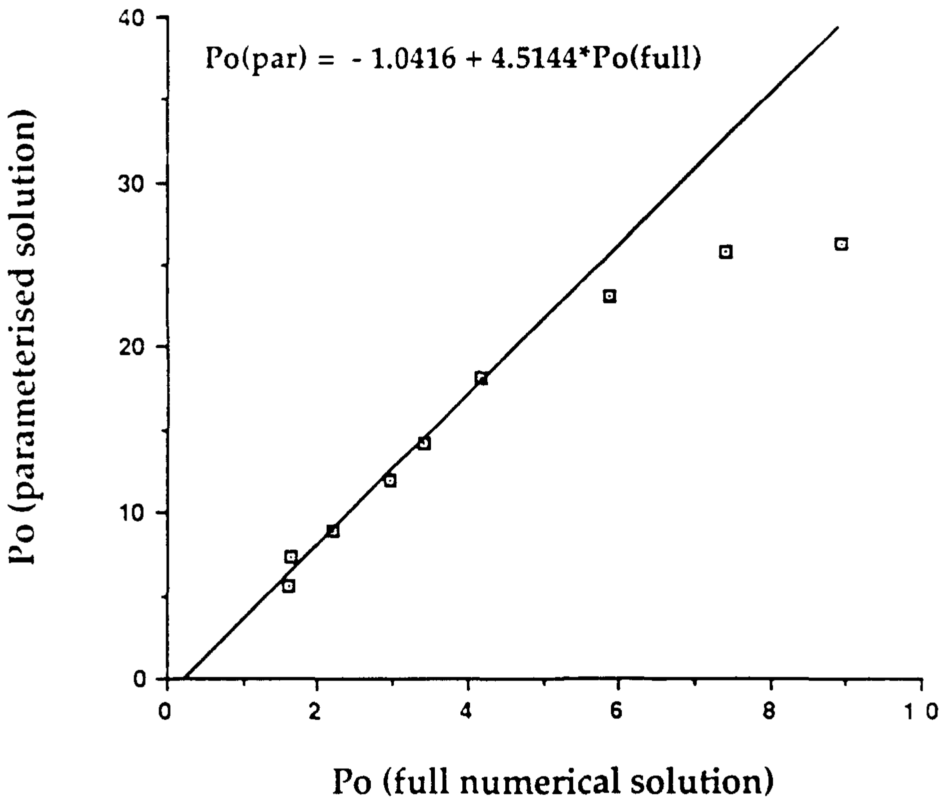

Figure 8 Relationship between the analysed input power P_0 for the parameterised model as a function of P_0 from the full numerical solution.

APPENDIX A

JUSTIFICATION OF THE LINEAR STRAIN APPROXIMATION

In this Appendix the condition under which it is valid to retain only the linear contribution to the strain field is derived. The starting point is the non-linear equation of motion for the particle displacement [4]

$$\rho_0 \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}\left[M_2 + M_3 \frac{\partial u}{\partial x} + M_4 \left(\frac{\partial u}{\partial x}\right)^2 + M_5 \left(\frac{\partial u}{\partial x}\right)^3 + \ldots\right] \tag{A1}$$

As indicated in Section 2, the strain field for one-dimensional propagation is [5]

$$S \equiv \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2 \tag{A2}$$

Defining

$$\zeta \equiv \frac{\partial u}{\partial x} \quad \text{then } \zeta = S - \frac{\zeta^2}{2}$$

$$\zeta^2 = \left(S - \frac{\zeta^2}{2}\right)^2$$

$$= \left(S^2 - S\zeta^2 + \frac{\zeta^4}{4}\right)$$

$$= S^2 - S\left(S - \frac{\zeta^2}{2}\right)^2 + \frac{\zeta^4}{4}$$

$$= S^2 - S\left(S^2 - S\zeta^2 + \frac{\zeta^4}{4}\right) + O(\zeta^4)$$

i.e.

$$\zeta^2 = S^2 - S^3 + (S\zeta)^2 + O(\zeta^4).$$

Then using $\zeta^2 = 2(S - \zeta)$ on the L.H.S. gives

$$\zeta = S - \frac{1}{2}S^2 + \frac{1}{2}S^3 + O(\zeta^4) \tag{A3}$$

The term in square brackets in Equation (A1) is then of a form, accurate to third order in the strain, given by

$$
\begin{aligned}
M_2 + M_3\zeta + M_4\zeta^2 &= M_2 + M_3\left(S - \frac{1}{2}S^2 + \frac{1}{2}S^3\right) + M_4\left(S^2 - S^3\right) \\
&= M_2 + M_3 S + \left(M_4 - \frac{M_3}{2}\right)S^2 + \left(\frac{M_3}{2} - M_4\right)S^3
\end{aligned} \tag{A4}
$$

Now,

$$
\begin{aligned}
&\frac{\partial^2 u}{\partial x^2} = \frac{\partial \zeta}{\partial x} = \frac{\partial S}{\partial x} - S\frac{\partial S}{\partial x} + \frac{3}{2}S^2\frac{\partial S}{\partial x} \\
&\therefore \frac{\partial^2 u}{\partial x^2}\left[M_2 + M_3\frac{\partial u}{\partial x} + M_4\left(\frac{\partial u}{\partial x}\right)^2 + \ldots\right] \\
&\approx \left(\frac{\partial S}{\partial x} - S\frac{\partial S}{\partial x} + \frac{3}{2}S^2\frac{\partial S}{\partial x}\right)\left(M_2 + M_3 S + \left(M_4 - \frac{M_3}{2}\right)S^2 + \ldots\right) \\
&= M_2\frac{\partial S}{\partial x} + \left(M_3 - M_2\right)S\frac{\partial S}{\partial x} + \left(\frac{3}{2}M_2 - \frac{3}{2}M_3 + M_4\right)S^2\frac{\partial S}{\partial x} + O\left(S^4\right).
\end{aligned} \tag{A5}
$$

Substituting Equation (A5) into the R.H.S. of Equation (A1) and differentiating with respect to x yields

$$
\rho_0\frac{\partial^2}{\partial t^2}\frac{\partial u}{\partial x} = M_2\frac{\partial^2 S}{\partial x^2} + \left(M_3 - M_2\right)\frac{\partial}{\partial x}\left(S\frac{\partial S}{\partial x}\right) + \left(\frac{3}{2}M_2 - \frac{3}{2}M_3 + M_4\right)\frac{\partial}{\partial x}\left(S^2\frac{\partial S}{\partial x}\right) \tag{A6}
$$

The term on the left hand side of Equation (A6) is, from Equation (A3), of the form

$$
\rho_0\frac{\partial^2}{\partial t^2}\left(S - \frac{1}{2}S^2 + \frac{1}{2}S^3\right) \tag{A7}
$$

Only the linear term in S need be retained since typically $S \sim 10^{-5}$, and as such the higher order terms are very small. Furthermore, if the material constants satisfy

$$
M_2 << M_3 << M_4 << \ldots \tag{A8}
$$

(as is usually the case if acoustic non-linearities are to be seen at all, due to the very small magnitude of the strain), then Equation (A6) can be expressed as

$$
\rho_0\frac{\partial^2 S}{\partial t^2} = M_2\frac{\partial^2 S}{\partial x^2} + M_3\frac{\partial}{\partial x}\left(S\frac{\partial S}{\partial x}\right) + M_4\frac{\partial}{\partial x}\left(S^2\frac{\partial S}{\partial x}\right) \tag{A9}
$$

Equation (A9) is identical to Equation (4) of Section 2, which is obtained directly from the approximation of using only the linear contribution to the strain field. The difference between Equation (4) and Equation (A9) is that the strain in Equation (A9) is the full, non-linear strain defined by Equation (A2). It is clear that the material constraints defined by Equation (A8) are a sufficient condition to allow use of the linearised expression for the strain field.

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UNCLASSIFIED
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1a. AR Number	1b. Establishment Number	2. Document Date	3. Task Number
AR-006-986	ERL-0643-RR	AUGUST 1992	DEF 89/210

4. Title

AN ANALYSIS OF THE EFFECT OF ACOUSTIC NON-LINEARITIES IN WIDE BANDWIDTH ACOUSTO-OPTIC BRAGG CELLS

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6. No. of Pages 26

7. No. of Refs. 9

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- [√] No Limitation
- [ ] Ref. by Author , Doc No. and date only.

14. DEFTEST Descriptors

Signal processing, Acousto-optics, Acoustic non-linearities*

15. DISCAT Subject Codes

1711

16. Abstract

In this report an extension of the theory of acoustic non-linearities accurate to fourth order is derived. A new fundamental limitation on the high frequency operation of acousto-optic signal processing architectures is demonstrated.

Page Classification
UNCLASSIFIED

16. Abstract (CONT.)

17. Imprint

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18. Document Series and Number	19. Cost Code	20. Type of Report and Period Covered
ERL-0643-RR	723238	RESEARCH REPORT

21. Computer Programs Used

22. Establishment File Reference(s)

23. Additional information (if required)